1 Given that $f(x)=(3x-5)^4$, find,

a) $f'(x)$,

[3 marks]

b) the gradient of the curve $y=f(x)$ at the point where $x=2$.

[1 mark]

2 Given that $f(x)=\sqrt{5x-1}$ find the value of $f'(1)$.

[4 marks]

3 A curve is defined such that $y=\frac{1}{2}\sin(6x)$, for $0 \le x \le \frac{\pi}{3}$.

Find the value(s) of x for all the horizontal tangents to the curve.

[5 marks]

4 Find the gradient of the curve $y=5e^{\frac{2}{5}x}$, at the point where $x=2$.

Give your answer as an exact number.

[6 marks]

5 Find the equation of the tangent to the function $f(x)=2\ln\left(\frac{1}{3}x-1\right)$ at the point where $x=6$. Give your answer in exact number form.

[6 marks]

6 Given that $y=\frac{\cos(2x)}{4}$,

a) find and expression for $\frac{dy}{dx}$,

[3 marks]

b) find the smallest positive x-value such $\frac{dy}{dx}=0$.

[3 marks]

7 A function is defined as $f(x)=(2x+3)^3$.

a) Find $f(1)$.

[1 mark]

b) Evaluate $f'(x)$.

[3 marks]

c) Calculate $f'(1)$.

[1 mark]

d) Evaluate the equation of the tangent to $f(x)$ at the point where $x=1$.

[3 marks]

8 Consider the function $f(x)=2e^{3x}+\frac{1}{2}\sin\left(\frac{3}{2}x\right)$.

a) Find $f'(x)$.

[5 marks]

b) Find $f'\left(\frac{\pi}{3}\right)$, giving your answer in exact form.

[2 marks]

9 Evaluate the gradient of the curve $y=2\sin(2x)-4\cos\left(\frac{1}{2}x\right)$ where $x=90°$.

[6 marks]

10 Consider the function $f(x)=5\ln\left(\frac{x}{2}\right)$ for $0<x<10$.

a) Sketch the graph of $y=f(x)$.

[3 marks]

b) Find $f(x)=0$.

[1 mark]

c) Find the gradient of the function at the point where $x=2$.

[3 marks]

d) Find the equation of the normal to the the function at the point where $x=2$.

[4 marks]

1 a) $f(x)=(3x-5)^4$

$y=u^4 \qquad \frac{dy}{du}=4u^3$

$u=3x-5 \qquad \frac{du}{dx}=3$

$f'(x)=3\times 4u^3$

$f'(x)=12(3x-5)^3$

[3 marks]

b) $f'(2)=12(3(2)-5)^3$

$f'(2)=12$

[1 mark]

2 $f(x)=\sqrt{5x-1}$

$y=u^{\frac{1}{2}} \qquad \frac{dy}{dx}=\frac{1}{\sqrt{u}}$

$u=5x-1 \qquad \frac{du}{dx}=5$

$f'(x)=\frac{5}{\sqrt{u}} \quad \Rightarrow \quad f'(x)=\frac{5}{\sqrt{5x-1}}$

$f'(1)=\frac{5}{\sqrt{5(1)-1}}$

$f'(1)=\frac{5}{2}$

[4 marks]

3

$$y=\frac{1}{2}\sin(6x)$$

$$y=\frac{1}{2}\sin u \qquad \frac{dy}{du}=\frac{1}{2}\cos u$$

$$u=6x \qquad \frac{du}{dx}=6$$

$$\frac{du}{dx}=6\times\frac{1}{2}\cos u \qquad \Rightarrow \qquad \frac{du}{dx}=3\cos(6x)$$

$$3\cos(6x)=0$$

$$6x=\frac{\pi}{2} \text{ or } 6x=\frac{3\pi}{2}$$

$$x=\frac{\pi}{12},\frac{\pi}{4}$$

[5 marks]

4

$$y=5e^{\frac{2}{5}x}$$

$$y=5e^{u} \qquad \frac{dy}{du}=5e^{u}$$

$$u=\frac{2x}{5} \qquad \frac{du}{dx}=\frac{2}{5}$$

$$\frac{dy}{dx}=\frac{2}{5}\times 5e^{u} \qquad \Rightarrow \qquad \frac{dy}{dx}=2e^{\frac{2}{5}x}$$

$$x=2 \qquad \frac{dy}{dx}=2e^{\frac{2}{5}(2)}$$

Gradient=$2e^{\frac{4}{5}}$

[6 marks]

5 $f(x)=2\ln\left(\frac{1}{3}x-1\right)$

$y=2\ln u \qquad \frac{dy}{du}=\frac{2}{u}$

$u=\frac{1}{3}x-1 \qquad \frac{du}{dx}=\frac{1}{3}$

$\frac{dy}{dx}=\frac{1}{3}\times\frac{2}{u} \quad \Rightarrow \quad \frac{dy}{dx}=\frac{2}{3\left(\frac{1}{3}x-1\right)}=\frac{2}{x-3}$

$f'(x)=\frac{2}{x-3}$ and $f'(6)=\frac{2}{6-3}=\frac{2}{3}$

$f(6)=2\ln\left(\frac{1}{3}(6)-1\right)=0$

$y=\frac{2}{3}x+c$

Use the point $(6,0)$ $\quad 0=\frac{2}{3}(6)+c \quad \Rightarrow \quad c=4$

$y=\frac{2}{3}x+4$

[6 marks]

6 a) $y=\frac{\cos(2x)}{4}$

$y=\frac{\cos u}{4} \qquad \frac{dy}{du}=\frac{-\sin u}{4}$

$u=2x \qquad \frac{du}{dx}=2$

$\frac{dy}{dx}=2\times\frac{-\sin u}{4} \quad \Rightarrow \quad \frac{dy}{dx}=\frac{-\sin(2x)}{2}$

[3 marks]

b) $\frac{dy}{dx} = \frac{-\sin(2x)}{2}$

$\frac{-\sin(2x)}{2} = 0$

$\sin(2x) = 0$

$2x = \pi \quad \text{as } x > 0$

$x = \frac{\pi}{2}$

[3 marks]

7 a) $f(x) = (2x+3)^3$

$f(1) = (2(1)+3)^3 = 125$

[1 mark]

b) $y = (2x+3)^3$

$y = u^3 \qquad \frac{dy}{du} = 3u^2$

$u = 2x+3 \qquad \frac{du}{dx} = 2$

$\frac{dy}{dx} = 2 \times 3u^2 \qquad \frac{dy}{dx} = 6(2x+3)^2$

$f'(x) = 6(2x+3)^2$

[3 marks]

c) $f'(1) = 6(2(1)+3)^2$

$f'(1) = 150$

[1 mark]

d) $y = 150x + c$

$(1,125) \quad 125 = 150 + c \quad \Rightarrow \quad c = -25$

$y = 150x - 25$

[3 marks]

8 a) $f(x) = 2e^{3x} + \frac{1}{2}\sin\left(\frac{3}{2}x\right)$

Split this into two parts:

First:

$y = 2e^{3x}$

$y = 2e^{u}$ $\qquad \frac{dy}{du} = 2e^{u}$

$u = 3x$ $\qquad \frac{du}{dx} = 3$

$\frac{dy}{dx} = 6e^{3x}$

Second:

$y = \frac{1}{2}\sin\left(\frac{3}{2}x\right)$

$y = \frac{1}{2}\sin u$ $\qquad \frac{dy}{du} = \frac{1}{2}\cos u$

$u = \frac{3}{2}x$ $\qquad \frac{du}{dx} = \frac{3}{2}$

$\frac{dy}{dx} = 3\cos\left(\frac{3}{2}x\right)$

Put both together:

$f'(x) = 6e^{3x} + 6e^{3x}3\cos\left(\frac{3}{2}x\right)$

[5 marks]

b) $f'\left(\frac{\pi}{3}\right) = 6e^{3\left(\frac{\pi}{3}\right)} + 3\cos\left(\frac{3}{2}\left(\frac{\pi}{3}\right)\right)$

$f'\left(\frac{\pi}{3}\right) = 6e^{\pi} + 3\cos\left(\frac{\pi}{2}\right)$

$f'\left(\frac{\pi}{3}\right) = 6e^{\pi}$

[2 marks]

9 Differentiate in two parts:

First:

$y = 2\sin(2x)$

$y = 2\sin u \qquad \frac{dy}{du} = 2\cos u$

$u = 2x \qquad \frac{du}{dx} = 2$

$\frac{dy}{dx} = 4\cos(2x)$

Second:

$y = 5\cos\left(\frac{1}{10}x\right)$

$y = 4\cos u \qquad \frac{dy}{du} = -4\sin u$

$u = \frac{1}{2}x \qquad \frac{du}{dx} = \frac{1}{2}$

$\frac{dy}{dx} = -2\sin\left(\frac{1}{2}x\right)$

Put together:

$\frac{dy}{dx} = 4\cos(2x) - \left(-2\sin\left(\frac{1}{2}x\right)\right)$

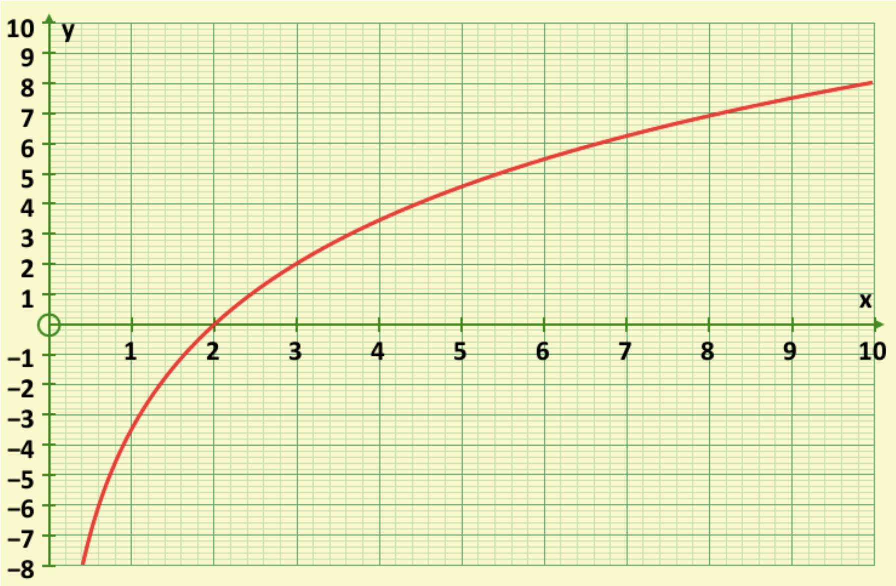
$\frac{dy}{dx} = 4\cos(2x) + 2\sin\left(\frac{1}{2}x\right)$

$x = 90, \quad \frac{dy}{dx} = -4 + \frac{4+\sqrt{2}}{2}$

$\text{Gradient} = \frac{4-\sqrt{2}}{2} \quad (=1.29)$

[6 marks]

10 a)



[3 marks]

b)

$$5\ln\left(\frac{x}{2}\right)=0$$

$$\ln\left(\frac{x}{2}\right)=0$$

$$x=2$$

[1 mark]

c)

$$f(x)=5\ln\left(\frac{x}{2}\right)$$

$$y=5\ln u \qquad \frac{dy}{du}=\frac{5}{u}$$

$$u=\frac{x}{2} \qquad \frac{du}{dx}=\frac{1}{2}$$

$$\frac{dy}{dx}=\frac{1}{2}\times\frac{5}{u} \quad \Rightarrow \quad \frac{dy}{dx}=\frac{5}{2\left(\frac{x}{2}\right)}=\frac{5}{x}$$

$$f'(x)=\frac{5}{x}$$

$$f'(2)=\frac{5}{2} \qquad \text{Gradient}=\frac{5}{2}$$

[3 marks]

d) At $x=2$ gradient$=\frac{5}{2}$ so normal is $-\frac{2}{5}$.

$y=c-\frac{2}{5}x$

$(2,0)\quad 0=c-\frac{4}{5} \quad\Rightarrow\quad c=\frac{4}{5}$

$y=\frac{4}{5}-\frac{2}{5}x$

[4 marks]